

① write $f(x) = \frac{2}{1+x^3}$ in sum form

$$\sum_{n=0}^{\infty} (-1)^n (2) (x^{3n})$$

Given Geo

① $f(x) = 3$

$$f'(x) = 1$$

$$f''(x) = 6$$

$$f'''(x) = -4$$

Taylor at $x=2$

$$3 + (x-2) + \frac{6(x-2)^2}{2} - \frac{4(x-2)^3}{3!}$$

$$3 + (x-2) + 3(x-2)^2 - \frac{2(x-2)^3}{3}$$

① $\sum_{n=0}^{\infty} \frac{(-1)^n 3x^n}{5(2)^n}$

Given Geometric

$$\frac{\frac{3}{5}}{1 + \frac{x}{2}}$$

$$\frac{6}{10+5x}$$

$$-2 < x < 2$$

Find $f(x)$ and interval

(2)

Approx $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ using

1st 3 term of Maclaurin.
(3 deci places)

$$\frac{\sqrt{3}}{3} - \frac{\left(\frac{\sqrt{3}}{3}\right)^3}{3} + \frac{\left(\frac{\sqrt{3}}{3}\right)^5}{5}$$

$$= \boxed{.526}$$

(2)

Find 1st 4 Term Maclaurin

$$f(x) = \frac{e^x + e^{-x}}{2}$$

$$\boxed{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}}$$

$$\frac{1}{2} \left(1+x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right. \\ \left. + 1-x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right)$$

(2)

$$f(x) = -x + \sin x$$

Find Maclaurin in sum form

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\frac{-x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$

(2)

$$f(x) = \frac{1}{2x+1} \quad \text{2nd order Taylor at } x=2$$

$$\frac{1}{5} - \frac{2}{25}(x-2) + \frac{8}{125} \frac{(x-2)^2}{2}$$

$$\boxed{\frac{1}{5} - \frac{2(x-2)}{25} + \frac{8(x-2)^2}{250}} \quad \frac{4}{125}$$

(2)

$$f(3) = 5$$

$$f'(3) = -2$$

$$f''(3) = 6$$

$$f'''(3) = -3$$

$$\text{Given } f^{(4)}(x) \leq \frac{4}{6}$$

Find Lagrange error bound of 3rd

order Taylor on $[2.9, 3.1]$

exact

$$\frac{6(1)^4}{4!}$$

$$= \boxed{.00025}$$

$$(2) \quad f(x) = x^3 + 3x^2 + 2x + 1$$

Approx $f(2.1)$ using 2nd order Taylor at $x=2$

$$f(2) = 1$$

$$f'(2) = 2$$

$$f''(2) = 6$$

$$1 + 2(x-2) + \frac{6(x-2)^2}{2}$$

$$\boxed{1.23}$$

③ Find sum

$$1 - \frac{\left(\frac{\pi}{3}\right)^2}{2!} + \frac{\left(\frac{\pi}{3}\right)^4}{4!} - \frac{\left(\frac{\pi}{3}\right)^6}{6!} + \dots$$

$$\left(\frac{1}{2}\right)$$

② Find 1st 3 Terms Maclaurin

$$\text{for } f(x) = \frac{4x}{1-x}$$

$$4x(1+x+x^2)$$

$$4x + 4x^2 + 4x^3$$

③ $f(x) = \ln x$

Find Lagrange Error

of 4th order Taylor at $x=2$
on $[1.5, 2.5]$

$$R_5(x) = \frac{\frac{24}{1.5^5} (.5)^5}{5!}$$

$$= .000823$$

$$f^{(5)}(x) = 24x^{-5} \quad M = \frac{24}{1.5^5} =$$

③ $f(x) = \cos(x^2)$

$$g'(x) = f(x)$$

$$g(0) = 3$$

Find 1st 4 Taylor at $x=0$
of $g(x)$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!}$$

$$\left\{ \cos(x^2) = 3 - \frac{x^4}{5 \cdot 2!} + \frac{x^8}{9 \cdot 4!} \right.$$

$$3 + x - \frac{x^5}{10} + \frac{x^9}{216}$$

③ Given Maclaurin for $f(x)$

$$= 3x^2 - 2x^3 + 4x^4 - 2x^5 \dots$$

find $f^{(4)}(0)$

$$\frac{f^{(4)}(0) x^4}{4!} = 4x^4$$

$$\frac{f^{(4)}(0)}{4!} = 4$$

$$f^{(4)}(0) = 96$$